

# PiXL Independence:

## Mathematics - Student Booklet

### KS5

#### Topic 1 – Basic Algebra

##### Contents:

- I. Basic Skills Check – 10 credits per skill check
- II. Short Exam Questions - 30 credits per section
- III. Further Practice – 30 credits each
- IV. Investigations – 80 credits each
- V. Academic stretch – 50 credits each

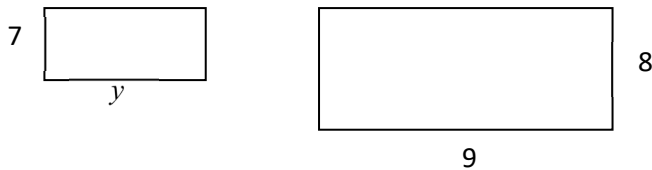
## I. Basic Skills Check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator.

### Skills Check 1

1. Write down the highest square factor of 50.
2. State whether true or false:  $\sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4}$

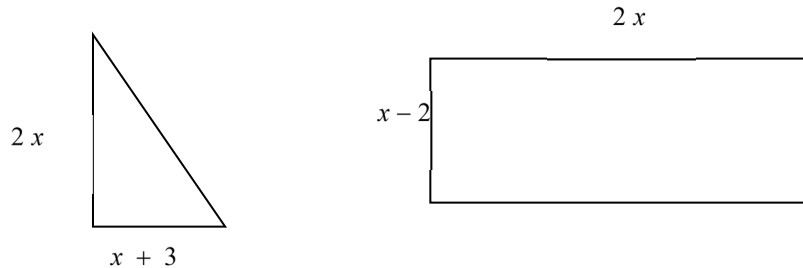
3. These two rectangles are similar. Find the value of  $y$ .



4. A solid cylinder has a base radius of  $r$  cm and a height of  $h$  cm. Write down a formula for the volume of the cylinder.
5. Find the distance between the two points  $(3, 4)$  and  $(9, 12)$ .
6. Solve the simultaneous equations  $y = 2x + 1$  and  $y = 7 - x$ .
7. Solve the equation  $6 - 3x = 2x + 1$
8. Expand the brackets  $(2x - 1)(3x + 2)$
9. Given  $f(x) = x^2 + 5x - 2$  find the value of  $f(4)$ .
10. Solve  $(x - 3)(2x + 5) = 0$ .

## Skills Check 2

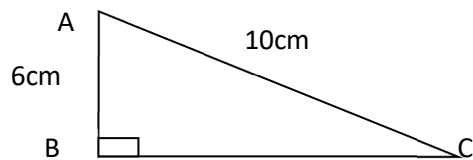
1. Write down two numbers that have a product of 24 and a sum of 11.
2. State whether true or false:  $\sqrt{100 \times 4} = \sqrt{400}$
3. Factorise  $x^2 - 2x - 35$ .
4. A solid cylinder has a base radius of  $r$  cm and a height of  $h$  cm. Write down a formula for the surface area of the cylinder.
5. Find the distance between the two points  $(-1, -2)$  and  $(4, 10)$ .
6. Solve  $x^2 + 15x + 56 = 0$ .
7. Simplify  $(\sqrt{2} + 1)(\sqrt{2} - 3)$ .
8. Solve  $3x(x - 3) = -4x - 2$
9. Solve the simultaneous equations
$$3x - 4y = 4$$
$$5x + 4y = 28$$
10. The triangle and the rectangle have the same area. Find the value of  $x$ .



### Skills Check 3

1. Write down the reciprocal of 2.
2. Write the equation of the line  $\frac{3}{4}x - 2 = \frac{1}{6}y$  in the form  $ax + by + c = 0$ .
3. Express  $\frac{6}{\sqrt{3}}$  in the form  $k\sqrt{3}$ .
4. Factorise  $x^2 + 7x$ .
5. Sketch the graph of  $y = \frac{1}{x}$ .
6. Solve the simultaneous equations
$$\begin{aligned}2y + 3x &= 18 \\5y - x &= 11\end{aligned}$$
7. Solve the inequality  $3x - 4 > 2$ .

8. In the given right-angled triangle ABC, find the length of BC.



9. Given  $P(x) = x^3 + 4x^2 + x - 6$  find the value of  $P(1)$ .
10. Write down the mid-point of the line joining  $(-2, -5)$  and  $(7, -3)$ .

## II. Short Exam Questions

### Section 1 - Quadratic Graphs.

1. Here are the equations of nine parabolas.

i.  $y = (x-3)(x-8)$

ii.  $y = 14 + 5x - x^2$

iii.  $y = 6x^2 - x - 70$

iv.  $y = x(4-x)$

v.  $y = (x+2)(x-8)$

vi.  $y = x^2 + 9x + 20$

vii.  $y = x^2 - 36$

viii.  $y = -3(x+3)(x+7)$

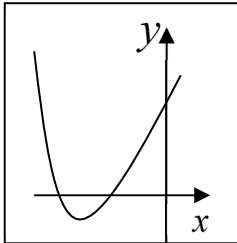
ix.  $y = x^2 + 4x + 4$

Answer the following questions, the answer can be 'none'

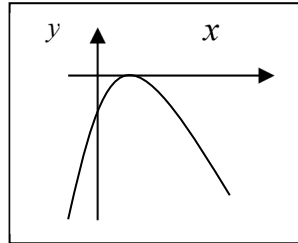
- A. Which of the parabolas cross the y-axis at a positive value of  $y$ ?
- B. For which of the parabolas is the vertex at the highest point of the graph?
- C. For which of the parabolas is the vertex to the left of the y-axis?
- D. Which of the parabolas pass through the origin?
- E. Which of the parabolas does not cross the x-axis at two separate points?
- F. Which of the parabolas has the y-axis as its axis of symmetry?
- G. Which two of the parabolas have the same axis of symmetry?
- H. Which of the parabolas have the vertex in the fourth quadrant?

2. Suggest a possible equation for each of the following graphs:

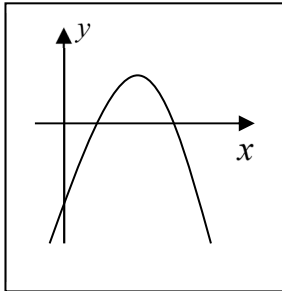
i.



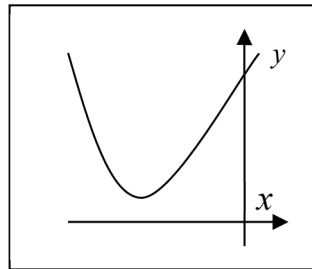
ii.



iii.



iv.



## Section 2 - Completing the Square and the Discriminant

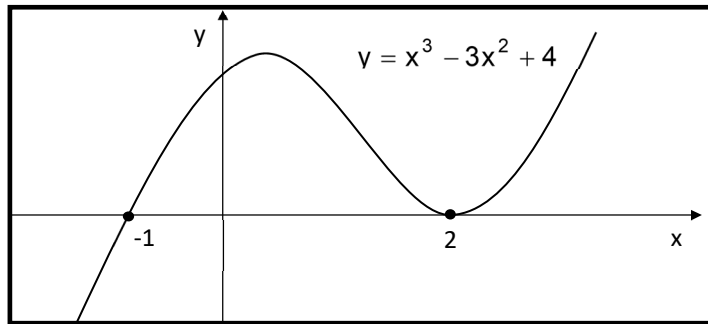
- By completing the square, solve the following quadratic equations, giving your answers in surd form where appropriate.
  - $x^2 - 10x - 24 = 0$
  - $x^2 - 4x - 1 = 0$
  - $10x^2 + 60x + 40 = 0$
- By completing the square, find in terms of  $k$  the roots of the equation  $x^2 + 2kx - 7 = 0$ . Prove that, for all real values of  $k$ , the roots of  $x^2 + 2kx - 7 = 0$  are real and different. Given that  $k = \sqrt{2}$ , find the exact roots of the equation.
- By first completing the square, explain why the equation  $x^2 + 10x + 27 = 0$  has no real roots.
- Show that  $f(x) = 3x^2 - 9x + 1$  can be written as  $f(x) = 3\left(x - \frac{3}{2}\right)^2 - \frac{23}{4}$ . What is the minimum value of  $f(x)$  and what is the value of  $x$  that makes  $f(x)$  a minimum?

### Section 3 - Inequalities and Graphs

- Given that the equation  $2x^2+4px+q=0$  has no real roots, show that  $2p^2 < q$ .
- The quadratic equation  $x^2+2kx+(k+56)=0$  has repeated roots.
  - Show that  $k^2 - k - 56 = 0$ .
  - Hence find the two possible values for  $k$ .
- Given that the quadratic equation  $x^2+(2m+3)x+m^2=0$  has real roots, show that  $m \geq -\frac{3}{4}$ .

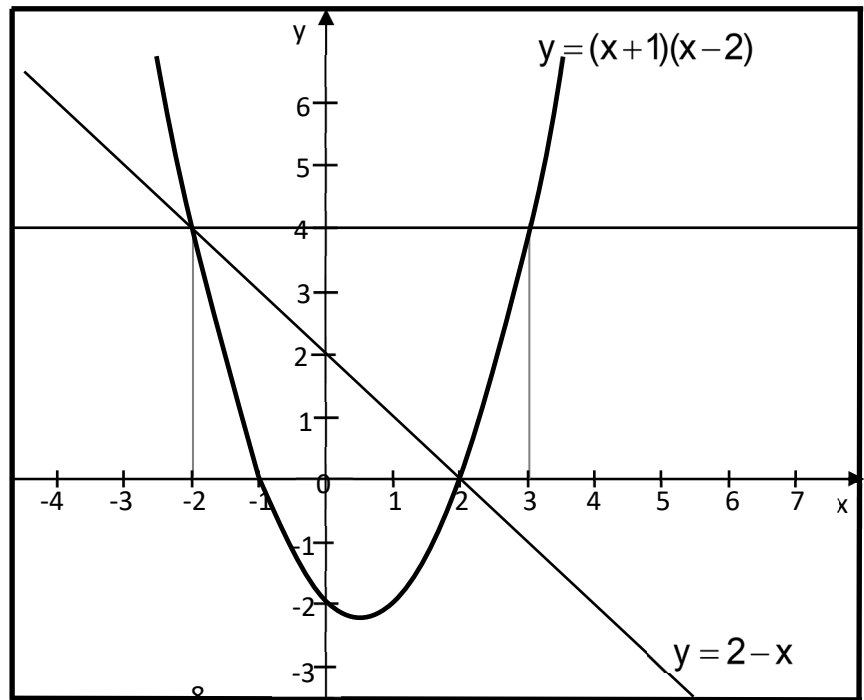
- Use the sketch shown opposite to write down the solutions to these inequalities:

- $x^3 - 3x^2 + 4 < 0$
- $x^3 - 3x^2 + 4 > 0$
- $x^3 - 3x^2 + 4 \leq 0$
- $x^3 - 3x^2 + 4 \geq 0$



- Use the graph shown opposite to write down the solutions to these inequalities:

- $(x+1)(x-2) < 0$
- $(x+1)(x-2) > 4$
- $(x+1)(x-2) \leq 2-x$





#### Section 4 - Mixed Questions

1. The straight line  $y - x = 2$  intersects the circle  $x^2 + y^2 + 5x = 9$  at the points A and B. Find the coordinates of A and B.
  
2. Write down
  - a) A rational number which lies between 4 and 5.
  - b) An irrational number which lies between 4 and 5.
  - c) A student says, 'When you multiply two irrational numbers together the answer is always an irrational number'.  
Simplify  $(2 + \sqrt{3})(2 - \sqrt{3})$  and comment on the student's statement.
  
3.
  - a) Express  $x^2 + 12x + 11$  in the form  $(x + a)^2 + b$ , finding the values of the constants  $a$  and  $b$ .
  - b) State the minimum value of the expression  $x^2 + 12x + 11$ .
  - c) Determine the values of  $k$  for which the quadratic equation  $x^2 + 3(k - 2)x + (k + 5) = 0$  has equal roots.
  
4.
  - a) Express  $x^2 + 4x - 5$  in the form  $(x + a)^2 + b$ , finding the values of the constants  $a$  and  $b$ .
  - b) Hence, or otherwise, find the range of values for which  $x^2 + 4x - 5 > 0$ .

### III. Further Practice

1. **Watch the video then complete the tasks at the end.**

<https://library.leeds.ac.uk/skills-algebra>

2. **Read the examples and make notes to help you complete the questions.**

You should then answer the questions when prompted to and mark your work. You should also complete the exercise at the bottom of the page and screen shot the marks you have achieved. Can you improve?

<http://www.cimt.org.uk/projects/mepres/step-up/sect2/index.htm>

3. **Watch the 'quadratics' section and complete the exam style questions.**

<https://www.examsolutions.net/a-level-maths/edexcel/c1-edexcel-tutorials/#Quadratic>

4. **Read the notes and attempt the questions.**

<http://furthermaths.org.uk/files/gcseextension/NA1.pdf>

<http://furthermaths.org.uk/files/gcseextension/NA8.pdf>

## IV. Investigations

For each of the following you should carry out the investigations then read the notes. You need to keep a detailed summary of what methods/approaches you have tried and what you then changed each time.

1. **Read the notes on the page and carry out the algebraic investigation. Complete the worksheet included.**

<https://www.teachmathematics.net/page/10900/consecutive-proofs>

2. **Can you find a sequence of consecutive integers that add up to 1000?** Try this investigation first, then read the following solution and make notes on the methods. What would you do differently? What did you change each time?

<https://ibmathsresources.com/2015/04/10/arithmatic-sequences-puzzle/>

3. **Investigate what makes numbers happy, or sad.**

<https://ibmathsresources.com/2014/06/19/friendly-numbers-solitary-numbers-perfect-numbers/>

<https://rich.maths.org/1314/index>

4. **Read the notes in the following article.** Make a summary of these notes and create a presentation/poster to explain the theory.

<https://ibmathsresources.com/2014/01/28/divisibility-tests-and-palindromic-numbers/>

5. **Risp**

<http://www.s253053503.websitehome.co.uk/risps/risp-3.pdf>

## V. Academic Reading

1. Read the following lecture notes and make a detailed summary.

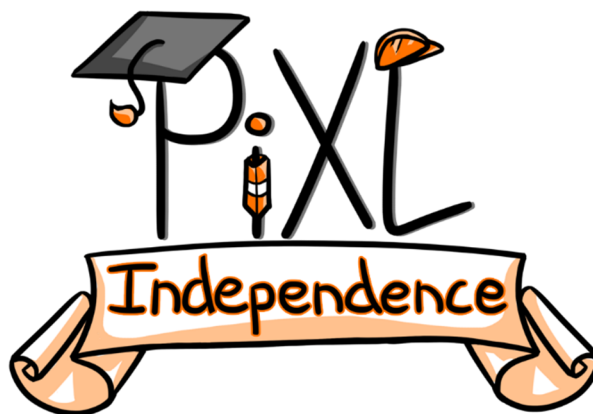
<https://www.maths.ox.ac.uk/system/files/attachments/lecture1.pdf>

2. Read the history of algebra article and make a detailed summary of the key points. Follow at least one of the links to extend your reading.

[http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Quadratic\\_etc\\_equations.html](http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Quadratic_etc_equations.html)

3. Complete module 1 - Advanced Problem Solving.

<https://nrich.maths.org/10209>



**Commissioned by The PiXL Club Ltd.**

This resource is strictly for the use of member schools for as long as they remain members of The PiXL Club. It may not be copied, sold, or transferred to a third party or used by the school after membership ceases. Until such time it may be freely used within the member school.

All opinions and contributions are those of the authors. The contents of this resource are not connected with, or endorsed by, any other company, organisation or institution.

PiXL Club Ltd endeavour to trace and contact copyright owners. If there are any inadvertent omissions or errors in the acknowledgements or usage, this is unintended and PiXL will remedy these on written notification.