

# PiXL Independence:

## Mathematics - Student Booklet

### KS5

## Topic 2 – Polynomials

### Contents:

- I. Basic Skills Check – 10 credits per skill check
- II. Short Exam Questions - 30 credits per section
- III. Further Practice – 30 credits each
- IV. Investigations – 80 credits each
- V. Academic Stretch – 50 credits each

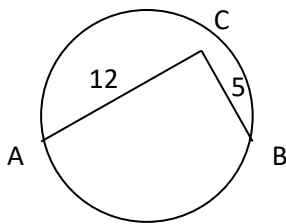
## I. Basic Skills Check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator.

10 credits for completing this quiz.

### Skills Check 1

1. Rewrite the equation of the straight line  $2x + 3y = 6$  in the form  $y = mx + c$ .
2. Factorise  $x^2 + 5x - 24$ .
3. Sketch the graph of  $y = x + 5$ .
4. Write  $\frac{1}{x^3}$  in the form  $x^n$ .
5. Show that the lines  $2x - 5y = 10$  and  $10y - 4x - 5 = 0$  are parallel.
6. Given  $f(x) = x^3 + 3x^2 - 6x - 8$ , find the value of  $f(2)$ .
7. Express  $(\sqrt{7} + 1)^2$  in the form  $a + b\sqrt{7}$ , where  $a$  and  $b$  are integers.
8. Solve the inequality  $5 < 2x - 1 < 17$ .
9. Find the length of AB, given X is the centre of the circle.



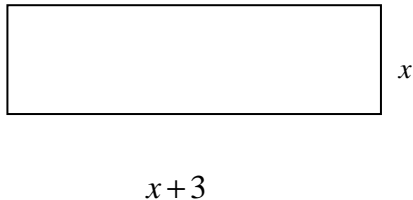
## **Skills Check 2**

1. Solve the inequality  $6(x+3) > 8 - 2(x+1)$ .
2. Sketch the graph of  $y = 3x^4$ .
3. Work out the point of intersection of the two lines  $x - 2y = 5$  and  $2x = 5y + 7$ .
4. Simplify  $6\sqrt{2} + 5\sqrt{8}$ .
5. Write down the mid-point of  $(4, 5)$  and  $(6, 3)$ .
6. Write  $\frac{1}{4x^3}$  in the form  $kx^n$ .
7. Solve the equation  $2x^2 + x - 4 = 0$ , leaving your answer in surd form.
8. It is given that  $f(x) = x^3 + 7x^2 + 8x + 10$ . Find the value of  $f(1)$  and  $f(-1)$ .
9. The points A and B have coordinates  $(12, 5)$  and  $(7, 3)$ . Find the gradient of AB.
10. Factorise  $x^2 - 9$ .

### Skills Check 3

1. Write down the reciprocal of  $\frac{1}{5}$ .
2. Given  $\frac{3+\sqrt{5}}{4+\sqrt{5}} = p + q\sqrt{5}$ , where  $p$  and  $q$  are rational numbers, find  $p$  and  $q$ .
3. Sketch the graph of  $y = -x^2$ .
4. Solve  $6x^2 + 11x + 3 = 0$  by factorisation.
5. Solve the inequality  $-3 \leq \frac{x}{2} \leq 5$ .
6. Given  $P(x) = 2x^3 + x^2 - 4x + 5$ , evaluate  $P(2)$ .
7. Write down the mid-point of  $(2, 10)$  and  $(-3, 0)$ .
8. Write down the gradient of the line joining  $(2, 10)$  and  $(-3, 0)$ .
9. Solve the simultaneous equations  $3x + 2y = 6$  and  $y = 5x - 10$ .

10.



The area of this rectangle is  $y \text{ cm}^2$  and the perimeter is  $y \text{ cm}$ . Find the dimensions of the rectangle.

## II. Short Exam Questions

### Section 1 - Algebraic Division

- Given that  $4x^3 - 25x^2 - 23x + 14 \equiv (x - 7)(px^2 + qx + r)$ , find the values of the constants  $p$ ,  $q$  and  $r$ .
- You are given that  $f(x) = x^3 + x^2 - 14x - 24$ ,
  - Write  $f(x)$  in the form  $(x + 2)(ax^2 + bx + c)$ .
  - By first factorising the quadratic part of your answer to (a), express  $f(x)$  as a product of three linear factors.
- You are given that  $g(x) = x^3 - 4x^2 - 7x + 10$ ,
  - Write  $g(x)$  in the form  $(x - 1)(ax^2 + bx + c)$ .
  - Hence express  $g(x)$  as a product of three linear factors.
  - Hence solve the equation  $g(x) = 0$ .
- The polynomial  $p(x)$  is defined by  $p(x) = 3x^3 - 29x^2 + 62x + 24$   
You are given that  $(x - 6)$  is a factor of  $p(x)$ .
  - Factorise  $p(x)$  completely.
  - Hence simplify  $\frac{2x^2 - 8x}{3x^3 - 29x^2 + 62x + 24}$ .
- The polynomial  $p(x)$  is defined by  $p(x) = 2x^3 + 5x^2 + x - 2$ .  
Given that  $(2x - 1)$  is a factor of  $p(x)$ .
  - Write  $p(x)$  as a product of three linear factors with integer coefficients.
  - Simplify the algebraic fraction  $\frac{3x^2 + 6x}{2x^3 + 5x^2 + x - 2}$  as far as possible.

## Section 2 - Factor Theorem

- Given that  $f(x) = x^3 - 7x - 6$ ,
  - Find  $f(1)$ ,  $f(-1)$ ,  $f(2)$ ,  $f(-2)$ ,  $f(3)$  and  $f(-3)$ .
  - Hence write  $f(x)$  as a product of three linear factors.
  - Solve the equation  $f(x) = 0$ .
- Given that  $g(x) = x^3 - 3x^2 - 6x + 8$ .
  - Use the factor theorem to show that  $(x-1)$  is a factor of  $g(x)$ .
  - Factorise  $g(x)$  completely.
  - Solve the equation  $g(x) = 0$ .
- Given that  $h(x) = x^3 - 3x^2 - 16x - 12$ .
  - Use the factor theorem to show that  $(x+2)$  is a factor of  $h(x)$ .
  - Write  $h(x)$  in the form  $(x + 2)(x^2 + px + q)$  where  $p$  and  $q$  are constants to be determined.
  - Solve the equation  $h(x) = 0$ , leaving your answers in surd form where appropriate.
- Given that  $g(x) = x^3 + ax + 6$ .
  - If  $(x+3)$  is a factor of  $g(x)$ , show that  $a = -7$ .
  - Hence solve the equation  $g(x) = 0$ , giving answers in **surd form** where appropriate.
- The function  $f(x) = x^3 + Ax^2 + Bx + 10$  has factors  $(x+2)$  and  $(x-5)$ .
  - Use this information to form and solve two simultaneous equations to find  $A$  and  $B$ .
  - Factorise  $f(x)$  completely.

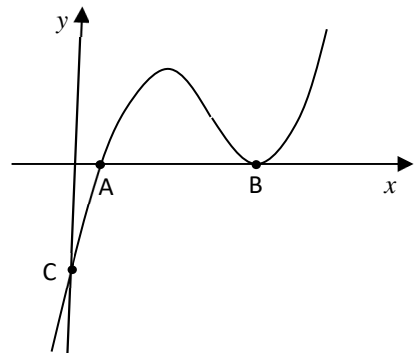
### Section 3 - Polynomials and Graphs

- Given that  $f(x) = 5x^3 - 6x^2 - 9x + 2$ .
  - Use the factor theorem to show that  $(x - 2)$  is a factor of  $f(x)$ .
  - Solve the equation  $f(x) = 0$ .
  - Hence sketch the graph  $y = f(x)$ , labelling the points where the curve crosses the coordinate axes.
  
- You are given that  $f(x) = (x + 1)(x + 2)(x - 2)$ .
  - Solve the equation  $f(x) = 0$ .
  - What does this tell you about the graph of  $y = f(x)$ ?
  - Where will the graph cross the y-axis?
  - Sketch the graph of  $y = f(x)$ .
  - Write the equation of the graph in the form  $y = ax^3 + bx^2 + cx + d$ .
  
- Solve the equation of  $(x + 4)(x - 2)(x - 1) = 0$ .
  - What does this tell you about the graph of  $y = (x + 3)(x + 2)(x - 1)$ ?
  - Where will the graph cross the y-axis?
  - Sketch the graph of  $y = (x + 4)(x + 2)(x - 1)$ .
  - Write the equation of the graph in the form  $y = ax^3 + bx^2 + cx + d$ .
  
- The sketch opposite shows the curve  $y = (2x - 3)(x - 5)^2$ .

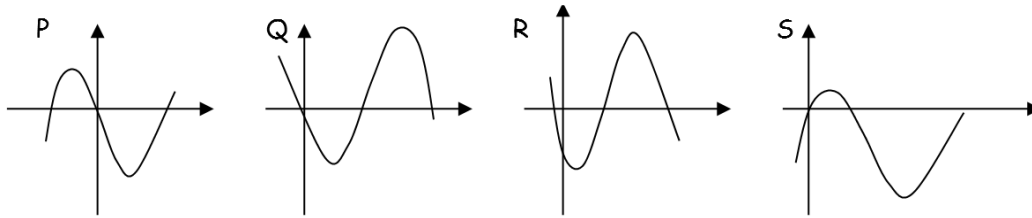
- The constants A, B and C on the sketch indicate the points where the curve meets the coordinate axes.

Write down the values of A, B and C.

- Write the equation of the graph in the form  $y = ax^3 + bx^2 + cx + d$ .



5. Show that  $x(x-4)(3-x)$  is equivalent to  $-x^3 + 7x^2 - 12x$ . Which of the graphs below is a sketch of  $y = -x^3 + 7x^2 - 12x$ ?



#### Section 4 - Mixed Questions

1.  $f(x) = x^3 + (a+1)x^2 - 18x + b$ , where  $a$  and  $b$  are integers.

Given that  $(x-4)$  is a factor of  $f(x)$ .

- (a) Show that  $16a + b + 8 = 0$ .

Given that  $(x+a)$  is also a factor of  $f(x)$ , and that  $a > 0$ .

- (b) Show that  $a^2 + 18a + b = 0$ .  
 (c) Hence find the value of  $a$  and the corresponding value of  $b$ .  
 (d) Factorise  $f(x)$  completely.

2. The polynomial  $P(x) = x^3 - 4x^2 + kx - 4$  leaves a remainder of  $-2$  when divided by  $(x-1)$ .
- Find the value of the constant  $k$ .
  - Show that  $(x-2)$  is **not** a factor of  $P(x)$ .

3. Sketch on a single diagram the following graphs:

- $y = x(x+3)(2-x)$ .
- $y = -\frac{3}{x}$ .
- Using your sketch, giving reasons, find the number of real solutions to the equation  $x(x+3)(2-x) + \frac{3}{x} = 0$ .

4. The function  $f(x) = x^3 + Ax^2 + Bx - 30$  has factors  $(x-2)$  and  $(x+5)$ .

- Use this information to form and solve two simultaneous equations to find  $A$  and  $B$ .
- Factorise  $f(x)$  completely.



### III. Further Practice

- 1. Watch the video and answer all the questions. Try the interactive resource at the bottom of the video. What do you notice about the graphs?**  
<https://www.examsolutions.net/tutorials/sketching-quadratic-graphs/?level=A-Level&board=Edexcel&module=C1&topic=1196>
- 2. Answer the questions and check your solutions.**  
[http://www.mesacc.edu/~scotz47781/mat120/notes/divide\\_poly/synthetic/synthetic\\_division\\_practice.html](http://www.mesacc.edu/~scotz47781/mat120/notes/divide_poly/synthetic/synthetic_division_practice.html)
- 3. Choose at least two of the sections in this unit to read and make notes on.**  
<http://www.coolmath.com/algebra/22-graphing-polynomials>
- 4. Assignment task to complete.**  
<http://tutorial.math.lamar.edu/ProblemsNS/Alg/Polynomials.aspx>
- 5. Create your own worksheet with at least ten polynomial multiplication problems and answer them.**  
<http://www.webmath.com/wsheel1.html>
- 6. Answer the problem solving questions.**  
[http://furthermaths.org.uk/manager\\_area/files/Problem%20solving%20series%2012.03Q Quadratics and Cubics 3rd edition.pdf](http://furthermaths.org.uk/manager_area/files/Problem%20solving%20series%2012.03Q%20Quadratics%20and%20Cubics%203rd%20edition.pdf)

## IV. Investigations

For each of the following you should carry out the investigations then read the notes. You need to keep a detailed summary of what methods/approaches you have tried and what you then changed each time.

- 1. Read the notes on the page and carry out the graph investigation. Complete the worksheet included, there's also an extension task to gain extra credit.**  
<https://www.teachmathematics.net/page/14038/exponential-graphs>
- 2. Euler's 9 Point Circle.** Read the article and follow the instructions in order to create a 9 point circle. Create a document/poster to instruct someone else on how to do this.  
<https://ibmathsresources.com/2017/09/30/eulers-9-point-circle/>
- 3. Read the article, follow some of the links. Can you write a summary of the key points and the maths involved?**  
<https://nrich.maths.org/2769>
- 4. Read the article and the comments. Which arguments are convincing? Are there flaws in the responses? What would your response be if you were to comment on the article?**  
<https://plus.maths.org/content/maths-minute-two-envelopes-problem>
- 5. Risp**  
<http://www.s253053503.websitehome.co.uk/risps/risp-8.pdf>

## V. Academic Reading

1. Read the following lecture notes and make a detailed summary.

<https://www.maths.ox.ac.uk/system/files/attachments/lecture2.pdf>

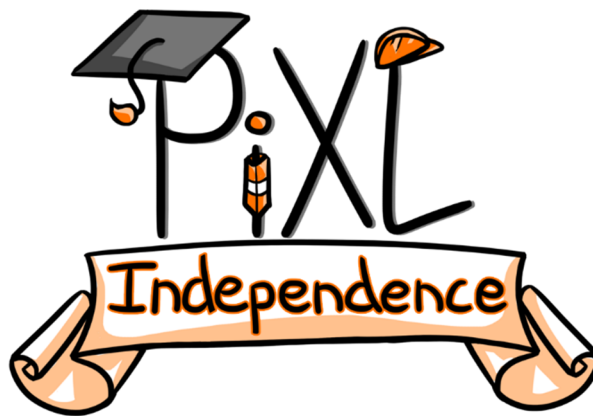
2. Read the history of algebra article and make a detailed summary of the key points. Follow at least one of the links to extend your reading.

[http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Fund theorem of algebra.html](http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Fund_theorem_of_algebra.html)

3. Complete STEP assignment 1/read the article to extend your understanding.

<https://maths.org/step/assignments/assignment-1>

<https://plus.maths.org/content/ringing-changes>



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