PiXL Independence:

Mathematics - Student Booklet

## KS5

Topic 3 – Coordinate Geometry

Contents:

I.   Basic Skills Check – 10 credits per skill check

II.   Short Exam Questions - 30 credits per section

III.   Further Practice – 30 credits each

IV.   Investigations – 80 credits each

V.   Academic Stretch – 50 credits each

I. Basic Skills Check

Answer the following questions. In order to improve your basic arithmetic you should attempt these without a calculator.

Skills Check 1

1. Sketch the graph of $y = \sqrt{x}$.

2. Write down the reciprocal of $-\dfrac{1}{10}$.

3. Describe the geometrical transformation which maps the parabola $y = x^2$ to $y = x^2 + 3$.

4. Solve $(2x - 3)^2 = 36$.

5. Write $-\dfrac{1}{5\sqrt{x}}$ in the form kx^n .

6. Solve the inequality $4(x - 2) \geq 7 - 2(x - 3)$.

7. State where the curve $y = (2x - 1)(x + 7)$ crosses the x -axis.

8. Small trays are to be made from rectangular pieces of card. Each piece of card is 8 cm by 5 cm and the tray is formed by removing squares of side x cm from each corner and folding the remaining card along the dotted lines, as shown in the diagram, to form an open-topped box. Write down a formula for the volume of this box.
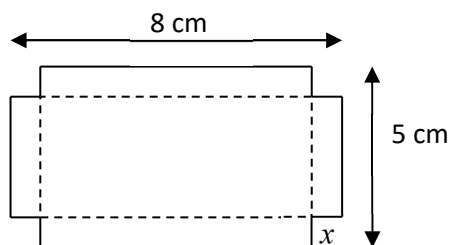

9. Express $\sqrt{8}\left(7 - \sqrt{2}\right)$ in the form $a + b\sqrt{2}$.

10. Write $x^2 + 10x + 30$ in the form $(x + A)^2 + B$ where A and B are values to be stated.

Skills Check 2

1. Write $x^2 - 4x + 5$ in the form $(x + A)^2 + B$ where A and B are values to be stated.

2. Solve the inequality $(x + 2)^2 \leq 5$.

3. Given $f(x) = x^3 + 12x^2 + 43x + 32$, find the value of $f(-1)$ and write down a factor of $f(x)$.

4. Sketch the graph of $y = (x - 2)^2 - 5$, showing clearly the coordinates of the vertex.

5. A (3, -4) and B (-5, 2) are the ends of a diameter of a circle. Write down the coordinates of the centre of the circle.

6. Given line A has equation $y = 3x + 7$, write down the equation of another line which is perpendicular to line A.

7. Simplify $\dfrac{\sqrt{11} + 1}{\sqrt{11} - 1}$.

8. Sketch the graph of $y = (x - 1)(2 - x)(2x + 1)$, showing clearly any intersections with the coordinate axes.

9. Factorise $-9x^2 + 18x - 5$.

10. Solve the simultaneous equations
$$y = 3x$$
$$xy + x = 2$$

Skills Check 3

1. Solve $(2x-3)^2 = 2x$, expressing your answers in surd form.

2. Solve the inequality $2x^2 < 18$.

3. Find the remainder when $f(x) = x^3 - 4x^2 - 5x + 7$ is divided by $(x+2)$.

4. Sketch the graph of $(x-1)^2 + (y+2)^2 = 25$.

5. Simplify $(2p^2qr)^2 \times 4pq^2r$.

6. Write down the reciprocal of –5.

7. Find the mid-point of the line joining (3, -5) and (5, 3).

8. Find the gradient of the line joining (3, -5) and (5, 3).

9. Factorise $12x^2 + 7x - 10$.

10. The diagram shows a square piece of card, with sides of length 6 cm. A smaller square of side x cm is cut from each corner as shown. The card is folded along the dotted lines to make an open box. Find an expression for the volume of the box in terms of x.

6 cm

6 cm

x
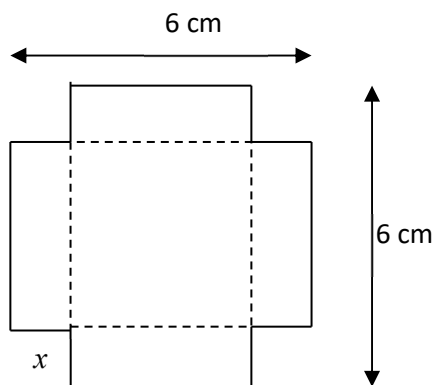
II.   Short Exam Questions

Section 1 - Equation of Lines

1. For each of the following equations give the **gradient** and the **y-intercept** of each line.
   a) y=2x-7
   b) y+3x=8
   c) 4y-2x+7=0

2. Find the equation of the line passing through the point (7, 10), and parallel to the line $8x + 4y = 3$.

3. Find the area of the triangle bounded by the coordinate axes and the line $3y - x + 12 = 0$.

4. Find the points at which the following lines cut the coordinate axes:
   a) x-3y+2 = 0
   b) 5x+9y-12 = 0

5. Find the equations of these straight lines:
   a) Gradient = 3, and passing through (2,5).
   b) Gradient = ⅔, and passing through (5,-1)
   c) Gradient = -4, and passing through (-2,13)

6. Find the equations of the line AB in each of the following cases:
   a) A(4,7) B(12,23)
   b) A(5,-1) B(3,5)
   c) A(3,10) B(3,27)
   d) A(-2,4) B(5,3)

7. Find the equation of the line passing through (6,-1) and parallel to the line 2x+3y = 4.

Section 2 - Midpoints, Distances and Bisectors

1. Find the mid-point of the lines joining these pairs of points:
   a) (3,8) to (11,2)
   b) (-5,13) to (8,-13)
   c) (-2,-7) to (0,-2)

2. For each of the following lines, find their gradient m_1 and the gradient of their perpendicular m_2:
   a) $y = 7x + 5$
   b) $4y - x + 12 = 0$
   c) $4x + 3y = 12$

3. Find the equation of the line through (4,2) and perpendicular to the line 5y = 2x+15.

4. Find the equation of the perpendicular bisector of the line segment AB with A(-4,3) and B(4,7).

5. Triangle ABC has vertices A(2,1), B(8,4), C(14,1). Prove the triangle is isosceles.

6. Find the equation of the perpendicular bisector of the line segment AB with A(6,9) and B(10,1).

7. The triangle PQR has corners P(1,1), Q(3,5) and R(5,-2).
   Find the equation of the line perpendicular to PQ and passing through R.

Section 3 - Intersecting Lines

1. Find the point of intersection of the following pairs of lines:
   a) $y = 3x - 1$ and $y = 5 - x$
   b) $y = x + 4$ and $3y - 2x + 8 = 0$
   c) $3x - 4y = 11$ and $5x + 2y = 14$
   d) $3x + 4y = 10$ and $y = \frac{1}{2}x - 5$

2. The sides of the triangle ABC are given by the following lines:

   AB: $x - 2y = -1$

   BC: $7x + 6y = 53$

   AC: $9x + 2y = 11$.

   a) Use simultaneous equations to find the coordinates of the three vertices of the triangle.
   b) Hence show that the triangle is isosceles.

3. The line *l* passes through the point (1,4) and is **perpendicular** to the line with the equation $2y = x - 7$.
   a) Find an equation for *l*.
   Find the coordinates of the point where the lines meet.

4. Given that…

   > The perpendicular bisector of the line joining any two points on the circumference of a circle will pass through the centre of the circle.

   Given that the points A(-2,4), B(6,4) and C(5,-3) all

   lie on the circumference of a circle:

   a) Write down the perpendicular bisector of the line segment AB.

   b) Work out the perpendicular bisector of the line segment AC.

   c) By solving simultaneously, find the centre of the circle.

   d) Calculate the radius of this circle.

<div align="center">
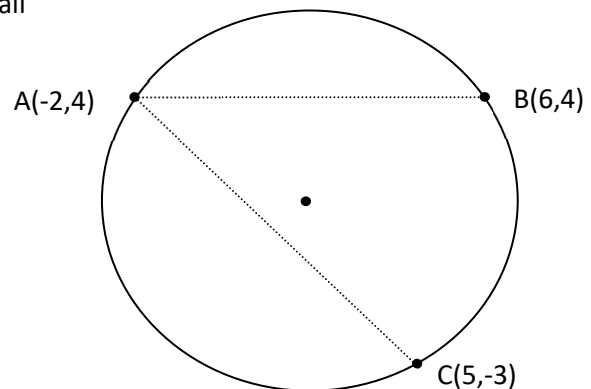
III.   Further Practice

</div>

1.   **Follow the lesson notes and make a summary of them. In particular the section about HOW to derive the formula.**
http://www.onlinemathlearning.com/coordinate-geometry.html

2.   **Answer the questions and check your solutions.**
http://pmt.physicsandmathstutor.com/download/Maths/A-level/C1/Topic-Qs/AQA/C1%20Coordinate%20Geometry.pdf

3.   **Watch the video for extra help.**
https://www.youtube.com/watch?v=c6MJ_Bb6bLI

4.   **Read all the notes, then scroll to the bottom and do the practice questions and the assignment problems.**
http://tutorial.math.lamar.edu/Classes/Alg/SolutionSets.aspx

IV. Investigations

For each of the following you should carry out the investigations then read the notes. You need to keep a detailed summary of what methods/approaches you have tried and what you then changed each time.

1. **Read the notes on the page and carry out the investigation.**
   https://www.teachmathematics.net/page/14809/even-and-odd-functions

2. **Modelling traffic.** Use the link in the following article as the starting point to investigate how to model traffic flow and how changing one variable will affect the traffic.
   https://ibmathsresources.com/2017/03/01/simulating-traffic-jams-and-asteroids/

3. **Read the article, follow some of the links. Can you write a summary of the key points and the maths involved?** Can you extend this further and investigate the maths behind crystals?

   https://nrich.maths.org/6574

4. **Watch the following video and make notes on how geometry has real world applications.**

   https://plus.maths.org/content/how-not-buckle-under-stress

5. **Follow the notes and questions for GAS GUZZLER.**
   http://www.nuffieldfoundation.org/fsmqs/level-3-algebra#Gas%20guzzlers

6. **Listen to three of the podcasts and make notes.**
   http://www.bbc.co.uk/programmes/b00srz5b/episodes/downloads

7. **Follow the notes and questions for the MAXIMUM AND MINIMUM PROBLEM.**
   http://www.nuffieldfoundation.org/fsmqs/level-3-algebra

8. **Risp.**
   http://www.s253053503.websitehome.co.uk/risps/risp-9.pdf

V.   Academic Reading

1.   **Read the following lecture notes and make a detailed summary.**

     https://www.maths.ox.ac.uk/system/files/attachments/lecture3.pdf

2.   **Read the history of algebra article and make a detailed summary of the key points. Follow at least one of the links to extend your reading.**
     http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/The_four_colour_theorem.html

3.   **Complete STEP assignment 1/read the article to extend your understanding.**

     https://maths.org/step/assignments/assignment-2

     https://plus.maths.org/content/circles-rolling-circles

4.   **Read the article and make notes on how this works.**

     https://plus.maths.org/content/maths-minute-polar-coordinates
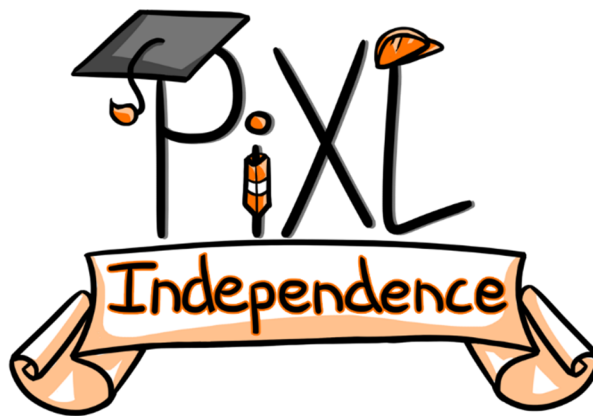
Commissioned by The PiXL Club Ltd.